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Abstract

The paper is devoted to the games of approach. We consider a controlled object whose dynamics is described by the linear differential system with pure time delay or the differential-difference system with commutative matrices in Euclidean space. The approaches to the solutions of these problems are proposed which based on the Method of Resolving Functions and the First Direct Method of L.S. Pontryagin. The guaranteed times of the game termination are found, and corresponding control laws are constructed. The results are illustrated by a model example.

Chapter 26
Quasi-Linear Differential-Deference Game of Approach

Lesia V. Baranovska

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26.1 Introduction

We consider the game problems of approach, which are central to the theory of conflict-controlled processes. They were the basis of the emergence of the theory, are the most informative and of considerable interest to researchers. The impetus for their development was given by real applications in economics, space technology, military affairs, biology, medicine, etc.

Conflict-controlled processes is a section of the mathematical control theory which is studying the manipulation of moving objects operated under in conditions of conflict and uncertainty. The evolution of an object can be described by systems of difference, ordinary differential, differential-difference, integral, integro-differential equations, systems of equations with distributed parameters, systems of equations with fractional derivatives, impulse influences and their various combinations (hybrid systems).

The term differential game is used for games in which the dynamics of an object is described by a system of ordinary differential equations. If the process is described by more complicated equations, possessing the semigroup property, then

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the term dynamic games is used. Finally, conflict-controlled processes are the most common term for determining the range of issues relating to game problems.

There are two types of dynamic games: games of degree and games of kind (see [1]). On the trajectory of the dynamical system, there is a function that depends on the initial state and on the player's control. In games of the first type, the goal of the first player is to minimize this function, set on the system trajectories, the purpose of the other one is to maximize it. In games of the second type, this functionality is the time of the exit of the trajectory of an object to a given terminal set, and the problem is to analyze the possibility of the pursuit of a trajectory of a system to a terminal set (the game of approach) or the deviation of the trap escape from this set (the deviation game).

The well-known pursuit strategies were mostly designed for military purposes. In practice, the rule of positional pursuit (see Fig. 26.1) and the rule of parallel pursuit (see Fig. 26.2) are widely used.

In the theory of differential games, along with the Pontryagin-Pshenichny's backward procedures (see [2, 3]), Krasovskii rule of extreme aiming (see [4]) and Isaacs's ideology (see [1]), there exist effective methods that constitutes share a separate direction.

Fig. 26.1 Positional pursuit

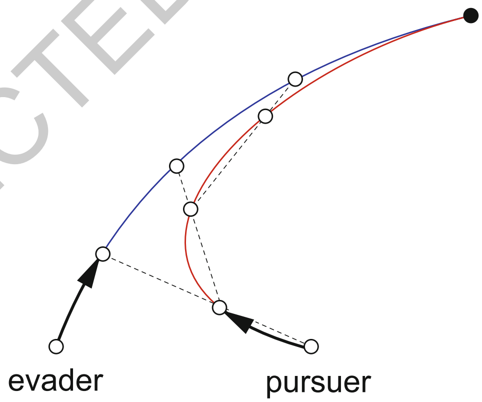
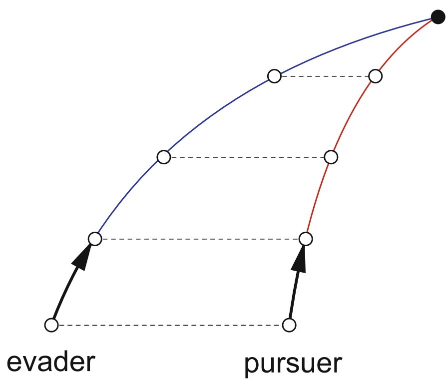


Fig. 26.2 Parallel pursuit



These are the First Direct Method of L.S. Pontryagin and the Method of Resolving Functions (see [5]). They are combined by the general principle of constructing controls of the pursuer on the basis of the Filippov-Castain multidimensional choice theorem (see [6]) and they provide a theoretical justification for the rule of parallel pursuit (see Fig. 26.2).

In this paper, the Method of Resolving Functions is chosen as the main tool for research, widely used to study conflict-controlled processes of various nature (see [5, 7]). The processes with fractional derivatives are studied in (see [8]), game problems of successive convergence are discussed in (see [9]), a general scheme of the method of resolving functions is given in (see [7]), the applied problem of soft meeting is solved in (see [10]), the nonstationary problems are considered in (see [11–14]), a variant of the matrix resolving functions are proposed in (see [15]), an approach games problem under the failure of controlling devices are considered in (see [16, 17]), and in (see [18, 19]) the cases of integral constraints on control are examined.

The future of many processes depends not only on the present state, but is also significantly determined by the entire prehistory. Numerous problems in the theory of automatic control, engineering, mechanics, radiophysics, biology, economics are described by differential equations with delay. For example, transport delay usually occurs in systems in which matter, energy or signals are transmitted over a distance (see [20]). In control systems, where one of the links is a person, the delay in the reaction of a person is important in constructing a mathematical model of the entire system. Distributed time delay occurs in the modeling of feeding systems and combustion chambers in a liquid monopropellant rocket motor with pressure feeding (see [21]). Great contribution to the development of these directions is made by Bellman R., Cooke K., Lunel S.M.V., Mitropolskii U.A., Myshkis A.D., Norkin S.B., Hale J.C., Azbelev N.V., Maksimov V.P., Rakhmatulina L.F. and others.

In (see [22–25]) the modification of the Method of Resolving Function for the differential-difference pursuit games is described, pursuit differential-difference games of approach with non-fixed time are considered in (see [26, 27]), system with time-varying delay is considered in (see [28]), in (see [29, 30]) the pursuit games with differential-difference equations of a neutral type are studied, an analytic approach based on the Method of Resolving Functions to study the differential-difference games of approach with commutative matrices is suggested in (see [31]), and the differential-difference games of approach for objects with different inertial are proposed in (see [32, 33]).

An attractive side of the Method of Resolving Functions is the fact that it allows us to effectively use modern technology of set-valued mappings and their selectors in the substantiation of game constructions and to obtain meaningful results on their basis (see [5]).

For dynamical systems whose evolution is described by differential-difference system with a cylindrical terminal set under the condition of L.S. Pontryagin introduces a resolving function, through which the game's end time is determined. The peculiarity of the basic scheme of the method is the fact that the time of the end of the game depends on a selector, the choice of which is in the power of the pursuer.

The resolving function characterizes the course of the game. When, at some point in time, the integral from it becomes a unit, this means that the trajectory falls onto the terminal set. Sufficient conditions for solvability of the problem of approach with a terminal set are provided. The pursuit process is divided into two stages.

On the first one $[0,\ t_*)$, where t_* is the moment of switching, the Method of Resolving Functions with using by the pursuer at the time t of the entire run-time control prehistory $v_t\,(\cdot)$ work. When at the instant t_* the integral of the resolving function turns into unity, the process of pursuit is switched to the First Direct Method of L.S. Pontryagin which is realized within the class of countercontrols in quasistrategy. In other words, from the moment of switching to the calculated moment, the ending of the game "stretches" time, and, in this area, the resolving function is considered to be zero, since it does not make any sense to accumulate it.

26.2 Differential-Difference Games of Approach with Commutative Matrices

Let $\mathbb{R}^n$ be an Euclidean space of points $z = (z_1, \ldots, z_n)$ and $K\,(\mathbb{R}^n)$ be a set of nonempty compacts in $\mathbb{R}^n$.

We consider the problem of approach for the system of differential-difference equations of retarded type (see [34–36]):

$$\dot{z}\,(t) = Az\,(t) + Bz\,(t-\tau) + \phi\,(u,\ v)\,, \quad z \in \mathbb{R}^n\,, \quad u \in U\,, \quad v \in V\,, \qquad (26.1)$$

where A and B are square constant matrices of order n; $U,\ V \in K\,(\mathbb{R}^n)$; $\phi\colon U \times V \to \mathbb{R}^n$, is jointly continuous in its variables; $\tau = \text{const} > 0$.

The phase vector consists of geometric coordinates, velocities and accelerations of the pursuer and the evader.

Let $z\,(t)$ be a solution of Eq. (26.1) under the initial condition

$$z\,(t) = z^0\,(t)\,, \quad -\tau \le t \le 0\,, \qquad (26.2)$$

where function $z^0\,(t)$ is absolutely continuous on $[-\tau\,,\ 0]$.

The piece of the trajectory $z^t\,(\,\cdot\,)$, where

$$z^t\,(\,\cdot\,) = \{\,z\,(t+s)\,,\ \ -\tau \le s \le 0\}$$

will be referred to as the state of system (26.1) at the moment t.

Definition 26.1 (See [37, 38]) For each $k = 1, 2, \ldots$, the time-delay exponential is defined as follows

$$exp_\tau\{B, t\} = \begin{cases} \Theta, \ -\infty < t < -\tau; \\ I, \ -\tau \le t < 0; \\ I + B\frac{t}{1!} + B^2\frac{(t-\tau)^2}{2!} + \cdots + B^k\frac{(t-(k-1)\tau)^k}{k!}, \ (k-1)\,\tau \le t \le k\tau, \end{cases}$$

where Θ is a zero matrix.

Lemma 26.1 (See [37, 38]) *Let $z(t)$ be a continuous solution to the system (26.1) with commutative matrices A and B under the initial condition in (26.2). Then,*

$$z(t) = exp\{A(t+\tau)\}exp_\tau\{B_1, t-\tau\}z^0(-\tau)$$
$$+\int_{-\tau}^{0} exp\{A(t-\tau)\}exp_\tau\{B_1, t-\tau-s\}[\dot{z}^0(s) - Az^0(s)]ds$$
$$+\int_0^t exp\{A(t-\tau-s)\}exp_\tau\{B_1, t-\tau-s\}\phi(u(s), v(s))\,ds,$$

or, in another form,

$$z(t) = F(t)\,a + \int_{-\tau}^{0} F(t-\tau-s)\,b(s)\,ds$$
$$+\int_0^t F(t-\tau-s)\,\phi(u(s), v(s))\,ds,$$

where we denote

$$a = exp\{A\tau\}z^0(-\tau), \quad b(t) = exp\{A\tau\}[\dot{z}^0(t) - Az^0(t)],$$

and matrix

$$F(t) = exp\{At\}exp_\tau\{B_1, t\},\ t \ge 0, \quad B_1 = exp\{-A\tau\}B,$$

is a solution to the similar system

$$\dot{z}(t) = Az(t) + Bz(t-\tau)$$

under the initial condition

$$F\left(t\right) \equiv exp\{At\}, \quad -\tau \le t \le 0.$$

Let us examine the differential-difference system (see [31]) as an example:

$$\dot{z}\left(t\right) = Az\left(t\right) + Bz\left(t-\tau\right) + u\left(t\right) - v\left(t\right), \quad z \in \mathbb{R}^{2n},$$

where

$$A = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix},$$

0 is a zero matrix, I is a unit matrix of order n,

$$U = \left\{ \begin{pmatrix} -u\left(t\right) \\ 0 \end{pmatrix} : u \in \mathbb{R}^n, \ ||u|| \le 2 \right\}, \quad V = \left\{ \begin{pmatrix} 0 \\ -v\left(t\right) \end{pmatrix} : v \in \mathbb{R}^n, \ ||v|| \le 1 \right\}.$$

The initial condition is equal to

$$z^0\left(t\right) = \left(z_1^0\left(t\right), \, z_2^0\left(t\right)\right), \quad -1 \le t \le 0.$$

We observe that matrices A and B are commutative, and $AB = BA = \Theta, \quad A^n = A, \quad B^n = B.$

From Lemma 26.1, we see that the functional matrix $F(t)$ is a solution to the similar system

$$\begin{pmatrix} F_{11}\left(t\right) & F_{12}\left(t\right) \\ F_{21}\left(t\right) & F_{22}\left(t\right) \end{pmatrix} \otimes I =$$

$$\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} F_{11}\left(t\right) & F_{12}\left(t\right) \\ F_{21}\left(t\right) & F_{22}\left(t\right) \end{pmatrix} \otimes I + \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} \cdot \begin{pmatrix} F_{11}\left(t-1\right) & F_{12}\left(t-1\right) \\ F_{21}\left(t-1\right) & F_{22}\left(t-1\right) \end{pmatrix} \otimes I =$$

$$\begin{pmatrix} F_{11}\left(t\right) & F_{12}\left(t\right) \\ F_{21}\left(t-1\right) & F_{22}\left(t-1\right) \end{pmatrix} \otimes I$$

and it satisfies the initial condition $F\left(t\right) \equiv exp\{At\}, \quad -\tau \le t \le 0$. Since

$$B_1 = exp\{-A\} \cdot B = \left(I_n - A + \frac{A^2}{2!} - \frac{A^3}{3!} + \cdots + (-1)^n \frac{A^n}{n!} + \cdots\right) \cdot B = B,$$

we obtain

$$
\begin{aligned}
F(t) &= exp\{At\} \cdot exp_{\tau}\{B,t\} \\
&= \left(I_n + At + A^2 \frac{t^2}{2!} + A^3 \frac{t^3}{3!} + \cdots + A^n \frac{t^n}{n!} + \cdots \right) \\
&\cdot \left(I_n + Bt + B^2 \frac{(t-1)^2}{2!} + B^3 \frac{(t-2)^3}{3!} + \cdots + B^n \frac{(t-(n-1))^n}{n!} + \cdots \right) \\
&= I_n + Bt + B^2 \frac{(t-1)^2}{2!} + B^3 \frac{(t-2)^3}{3!} + \cdots + B^n \frac{(t-(n-1))^n}{n!} + \cdots \\
&+ At + A^2 \frac{t^2}{2!} + A^3 \frac{t^3}{3!} + \cdots + A^n \frac{t^n}{n!} + \cdots = \begin{pmatrix} e^t & 0 \\ 0 & F_{22}(t) \end{pmatrix} \otimes I,
\end{aligned}
$$

where

$$
F_{22}(t) = exp_1\{I,t\} = 1 + \frac{t}{1!} + \frac{(t-1)^2}{2!} + \frac{(t-2)^3}{3!} + \cdots + \frac{(t-(k-1))^k}{k!},
$$
$$
(k-1) \le t \le k, \;\; k = 0, 1, 2, \ldots .
$$

The terminal set has cylindrical form, i.e.

$$
M^* = M_0 + M, \tag{26.3}
$$

where M_0 is a linear subspace in $\mathbb{R}^n$ and M is a compact set from the orthogonal complement of M_0 in $\mathbb{R}^n$.

The players choose their controls in the form of certain functions. Thus, the pursuer and the evader affect the process (26.1), pursuing their own goals. The goal of the pursuer (u) is in the shortest time to bring a trajectory of the process to a certain closed set M^*; the goal of the evader (v) is to avoid a trajectory of the process from meeting with the terminal set (26.3) on a whole semi-infinite interval of time or if is impossible to maximally postpone the moment of meeting.

Now we describe what kind of information is available to the pursuer in the course of the game.

Denote by Ω_U, Ω_V the sets of Lebesgue measurable functions $u(t)$, $v(t)$, $u(t) \in U$, $v(t) \in V$, $t \ge 0$, respectively. A mapping that puts into correspondence to a state $z^0(\cdot)$ some element in Ω_V is called an open-loop strategy of the evader, specific realization of this strategy for a given initial state $z^0(\cdot)$ of process (26.1) is called an open-loop control. In the process of the game (26.1), (26.3), the evader applies open-loop controls $v(\cdot) \in \Omega_V$.

Function

$$
u(t) = u\left(z^0(\cdot),\ t,\ v(t)\right),
$$

such that $v(\cdot) \in \Omega_V$ implies $u(\cdot) \in \Omega_U$ is called countercontrol (stroboscopic strategy of Hajek (see [39])) of pursuer corresponding to initial state $z^0(\cdot)$. The game is evolving on the closed time interval $[0, T]$. We assume that the pursuer chooses his control in the form

$$u(t) = u\left(z^0(\cdot),\ t,\ v_t(\cdot)\right), \quad t \geq 0,$$

where $v_t(\cdot) = \{v(s) : s \in [0, t],\ v(\cdot) \in \Omega_V\}$, and $u(\cdot) \in \Omega_U$.

Under these hypotheses, we will play the role of the pursuer and find sufficient conditions on the parameters of the problem (26.1), (26.3), insuring the game termination for certain guaranteed time.

Let π be the orthogonal projector from $\mathbb{R}^n$ onto the subspace L. Consider the set-valued mapping

$$W(t, v) = \pi F(t)\phi(U, v), \quad W(t) = \bigcap_{v \in V} W(t, v),$$

where $F(t)$ is defined in Lemma 26.1.

Condition 1 (Pontryagin's Condition) The mapping $W(t) \neq \emptyset$ for all $t \geq 0$.

Remark 26.1 For the linear process $(\phi(u, v) = u - v)$

$$W(t) = \pi K(t) U \overset{*}{-} \pi K(t) V,$$

where $\overset{*}{-}$ is a geometric subtraction of the sets (Minkowski' difference) (see [40]).

By virtue of the assumptions on the process parameters, the set-valued mapping $W(t, v)$ is continuous on the set $[0, +\infty) \times V$ in Hausdorff metric. Consequently, as follows from Condition 1, the mapping $W(t)$ is upper semi-continuous and therefore Borel measurable function (see [41]). Hence, there exists at least one Borelian selection $g(t)$, $g(t) \in W(t)$, $t \geq 0$ (see [42]). Let us denote by $G = \{g(\cdot) : g(t) \in W(t),\ t \geq 0\}$ the set of all Borelian selections of the set-valued mapping $W(t)$. For fixed $g(\cdot) \in G$ we put

$$\xi\left(t,\ z^0(\cdot),\ g(\cdot)\right) =$$
$$= \pi F(t) a + \int_{-\tau}^{0} \pi F(t - \tau - s) b(s) ds + \int_0^t g(s) ds,$$

and consider the resolving function

$$\alpha\left(t, s, z^0(\cdot), m, v, g(\cdot)\right) = \alpha_{W(t-\tau-s, v) - g(t-\tau-s)}\left(m - \xi\left(t,\ z^0(\cdot),\ g(\cdot)\right)\right)$$

for $t \geq s \geq 0$, $v \in V$, $m \in M$, $x \in \mathbb{R}^n$.

By virtue of the properties of the superposition of set-valued mappings and functions, it is Borel measurable function in s, v (see [5]). Finally, denote

$$\alpha\left(t, s, z^0(\,\cdot\,), v, g(\,\cdot\,)\right) = \max_{m\in M} \alpha\left(t, s, z^0(\,\cdot\,), m, v, g(\,\cdot\,)\right),$$

and then we obtain the resolving function

$$\begin{gathered}\alpha\left(t, s, z^0(\cdot), v, g(\cdot)\right) = \sup\{\alpha \geq 0\ : \\ [W(t-\tau-s, v) - g(t-\tau-s)] \cap \alpha\left[M - \xi\left(t, z^0(\cdot), g(\cdot)\right)\right] \neq \emptyset\}.\end{gathered} \tag{26.4}$$

Moreover, we also observe that function $\alpha\left(t,\ s,\ z^0(\,\cdot\,),\ v,\ g(\,\cdot\,)\right) = +\infty$ for all $s \in [0\,,\ t]\,,\ v \in V$, if and only if $\xi\left(t,\ z^0(\,\cdot\,),\ g(\,\cdot\,)\right) \in M$. If for some $t \geq 0$ $\xi\left(t,\ z^0(\,\cdot\,),\ \gamma(\,\cdot\,)\right) \notin M$, then function (26.4) assumes finite values.

Define the function T by

$$\begin{gathered}T = T\left(z^0(\,\cdot\,),\ g(\,\cdot\,)\right) \\ = \inf\left\{t \geq 0\ :\ \int_0^t \inf_{v\in V} \alpha\left(t\,,\ s\,,\ z^0(\,\cdot\,)\,,\ v,\ g(\,\cdot\,)\right) ds \geq 1\right\}, \qquad g(\,\cdot\,) \in G.\end{gathered} \tag{26.5}$$

If the inequality in the curly brackets is not satisfied for all $t \geq 0$, we set $T\left(z^0(\,\cdot\,)\,, g(\,\cdot\,)\right) = +\infty$.

Theorem 26.1 *Let the conflict controlled process (26.1), (26.3)) with the initial condition (26.2) and commutative matrices A and B satisfy Condition 1, and let the set M be convex, for the given initial state* $z^0(\,\cdot\,)$ *and some selection* $g^0(\,\cdot\,) \in G$ $T = T\left(z^0(\,\cdot\,)\,,\ g^0(\,\cdot\,)\right) < +\infty$.

Then a trajectory of the process (26.1), (26.3) can be brought by the pursuer from $z^0(\,\cdot\,)$ *to the terminal set* M^* *at the moment* T *under arbitrary admissible controls of the evader.*

Proof Let $v(\,\cdot\,) \in \Omega_V$. First consider the case when $\xi\left(T,\ z^0(\,\cdot\,),\ g^0(\,\cdot\,)\right) \notin M$. We introduce the controlling function

$$\begin{gathered}h(t) = h\left(T,\ t,\ s,\ z^0(\,\cdot\,)\,, v(\,\cdot\,)\,,\ g^0(\,\cdot\,)\right) \\ = 1 - \int_0^t \alpha\left(T,\ s,\ z^0(\,\cdot\,)\,,\ v(s)\,,\ g^0(\,\cdot\,)\right) ds, \qquad t \geq 0.\end{gathered}$$

From the definition of time T, there exists a switching time $t_* = t_*(v(\,\cdot\,))\,,\quad 0 < t_* \leq T$, such that $h(t_*) = 0$.

Let us describe the rules by which the pursuer constructs his control on the so-called active and the passive parts, $[0\,,\ t_*)$ and $[t_*\,,\ T]\,,$ respectively.

Consider the set-valued mapping

$$U_1(s, v) = \Big\{u \in U\ :\ \pi F\,(T-\tau-s)\,\phi\,(u,\ v) - g^0\,(T-\tau-s)$$
$$\in \alpha\left(T,\ s,\ z^0\,(\cdot)\,,\, v\,(s)\,,\, g^0\,(\cdot)\right)\left[M - \xi\left(T, z^0\,(\cdot)\,,\, g^0\,(\cdot)\right)\right]\Big\}.$$

From assumptions concerning the process (26.1), (26.3) parameters, with account of properties of the resolving function, it follows that the mapping $U_1\,(s,\ v)$ is a Borel measurable function on the set $[0,\ T] \times V$. Then selection

$$u_1\,(s,\ v) = \text{lex}\min U_1\,(s,\ v)$$

appears as a jointly Borel measurable function in its variables (see [41]). The pursuer's control on the interval $[0,\ t_*)$ is constructed in the following form

$$u\,(s) = u_1\,(s,\ v\,(s))\,,$$

being superposition of Borel measurable functions it is also Borel measurable function (see [41]).

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being superposition of Borel measurable functions it is also Borel measurable function (see [41]).

Set

$$\alpha\left(T,\ s,\ z^0\,(\,\cdot\,)\,,\ v\,(s)\,,\ g^0\,(\,\cdot\,)\right) \equiv 0, \qquad s \in [t_*,\ T]\,.$$

Then the mapping

$$U_2\,(s,\ v)$$
$$= \Big\{u \in U : \pi F\,(T-\tau-s)\,\phi\,(u,\ v) - g^0\,(T-\tau-s) = 0\Big\}\,, \qquad s \in [t_*,\ T]\,,\ v \in V$$

is Borel measurable function in its variables, and its selection

$$u_2\,(s,\ v) = \text{lex}\min U_2\,(s,\ v)$$

is Borel measurable function also.

On the interval $[t_* ,\ T]$ we set the pursuer's control equal to

$$u(s) = u_2(s,\ v(s))\ . \tag{26.6}$$

It is measurable function too (see [4, 9]).

Let $\xi\left(T,\ z^0(\,\cdot\,),\ g^0(\,\cdot\,)\right) \in M$. In this case, we choose the pursuer's control on the interval $[0,\ T]$ in the form (26.6).

Thus, the rules are defined, to which the pursuer should follow in constructing his control. We will now show that if the pursuer follows these rules in the course of the game, a trajectory of process (26.1) hits the terminal set at the time T under arbitrary admissible controls of the evader.

By virtue of Lemma 26.1, the Cauchy formula for the system (26.1) implies the representation

$$\begin{aligned}\pi z(T) = \pi F(T)\,a + \int_{-\tau}^{0} \pi F(T-\tau-s)\,b(s)\,ds \\ + \int_0^T \pi F(T-\tau-s)\,\phi(u(s),\ v(s))\,ds.\end{aligned} \tag{26.7}$$

First we examine the case when $\xi\left(T,\ z^0(\,\cdot\,),\ g^0(\,\cdot\,)\right) \notin M$.

By adding and subtracting from the right-hand side of Eq. (26.7) the value $\int_0^T g^0(T-\tau-s)\,\mathrm{d}s$, one can deduce

$$\begin{aligned}&\pi z(T)\\ &= \left[\pi F(T)\,a + \int_{-\tau}^{0} \pi F(T-\tau-s)\,b(s)\,\mathrm{d}s + \int_0^T g^0(T-\tau-s)\,\mathrm{d}s\right]\\ &\quad + \int_0^T \left[\pi F(T-\tau-s)\,\phi(u(s),\ v(s)) - g^0(T-\tau-s)\right]\mathrm{d}s\\ &\in \xi\left(T,\ z^0(\,\cdot\,),\ g^0(\,\cdot\,)\right) +\\ &\int_0^T \alpha\left(T,\ s,\ z^0(\,\cdot\,),\ v,\ g^0(\,\cdot\,)\right)[M - \xi\left(T,\ z^0(\,\cdot\,),\ g^0(\,\cdot\,)\right)]\mathrm{d}s\\ &= \xi\left(T,\ z^0(\,\cdot\,),\ g^0(\,\cdot\,)\right) + \int_0^T \alpha\left(T,\ s,\ z^0(\,\cdot\,),\ v,\ g^0(\,\cdot\,)\right) M\,\mathrm{d}s\\ &\quad - \int_0^T \alpha\left(T,\ s,\ z^0(\,\cdot\,),\ v,\ g^0(\,\cdot\,)\right)\xi\left(T,\ z^0(\,\cdot\,),\ g^0(\,\cdot\,)\right)\mathrm{d}s.\end{aligned} \tag{26.8}$$

By virtue (26.8) and $\alpha\left(T,\ s,\ z^0(\cdot),\ v(s),\ g^0(\cdot)\right)=0$, $s\in[t_*,\ T]$ we have the inclusion

$$\pi z(T)\in\xi\left(T,\ z^0(\cdot),\ g^0(\cdot)\right)\left[1-\int_0^{t_*}\alpha\left(T,\ s,\ z^0(\cdot),\ v(s),\ g^0(\cdot)\right)\mathrm{d}s\right]+\int_0^{t_*}\alpha\left(T,\ s,\ z^0(\cdot),\ v(s),\ g^0(\cdot)\right)M\mathrm{d}s.$$

Since $\int_0^{t_*}\alpha\left(T,\ s,\ z^0(\cdot),\ v(s),\ g^0(\cdot)\right)\mathrm{d}s=1$ and the set M is convex then $\pi z(T)\in M$. Then, applying the rule of the pursuer control for the case when $\xi\left(T,\ z^0(\cdot),\ g^0(\cdot)\right)\in M$, we obtain the inclusion $\pi z(T)\in M$. The proof is therefore complete.

Corollary 26.1 *Assume that the differential-difference game of approach (26.1), (26.3) is linear ($\phi(u,\ v)=u-v$), matrices A and B are commutative, Condition 1 holds, there exists a continuous positive function $r(t)$, $r:\mathbb{R}\to\mathbb{R}$, and a number $l\geq 0$ such that $\pi F(t)U=r(t)S$, $M=lS$, where S is the unit ball centered at zero in the subspace L.*

Then when $\xi\left(t,\ z^0(\cdot),\ g(\cdot)\right)\notin lS$, the resolving function (26.4) is the largest root of the quadratic equation for $\alpha>0$

$$\left\|\pi F(t-\tau-s)v+g(t-\tau-s)-\alpha\xi\left(t,\ z^0(\cdot),\ g(\cdot)\right)\right\|==r(t-\tau-s)+\alpha l. \tag{26.9}$$

Proof By virtue of the assumptions of Corollary 26.1, we conclude from expression (26.4) that the resolving function $\alpha\left(T,\ s,\ z^0(\cdot),\ v,\ g(\cdot)\right)$ for fixed values of its arguments is the maximal number α such that

$$[r(t-\tau-s)S-\pi F(t-\tau-s)v-g(t-\tau-s)]\cap\alpha\left[lS-\xi\left(t,\ z^0(\cdot),\ g(\cdot)\right)\right]\neq\emptyset.$$

The last expression is equivalent to the inclusion

$$\pi F(t-\tau-s)v+g(t-\tau-s)-\alpha\xi\left(t,\ z^0(\cdot),\ g(\cdot)\right)\in[r(t-\tau-s)+\alpha l)]S.$$

Due to the linearity of the left-hand side of this inclusion in α, the vector $\pi F(t-\tau-s)v+g(t-\tau-s)-\alpha\xi\left(t,\ z^0(\cdot),\ g(\cdot)\right)$ lies on the boundary of the ball $[r(t-\tau-s)+\alpha ql]S$ for the maximal value of α. In other words, the length of this vector is equal to the radius of this ball that is demonstrated by (26.9). The proof is complete.

26.3 Differential-Difference Games of Approach with Pure Time Delay

We consider the problem of approach, which is described by the system of differential-difference equations with pure time delay (see [38, ~~40, 41~~])

$$\dot{z}\,(t) = Bz\,(t-\tau) + \phi\,(u,\ v)\,, \quad z \in \mathbb{R}^n, \quad u \in U, \quad v \in V, \quad t \geq 0, \tag{26.10}$$

with the initial condition (26.2).

Lemma 26.2 (See [43]) *Let $z\,(t)$ be a continuous solution to the system (26.10) under the initial condition (26.2). Then,*

$$z\,(t) = exp_\tau\{B, t\}z^0\,(-\tau) + \int_{-\tau}^{0} exp_\tau\{B, t-\tau-s\}\dot{z}^0\,(s)\,ds$$
$$+ \int_0^t exp_\tau\{B, t-\tau-s\}\phi\,(u\,(s)\,, v\,(s))\,ds.$$

The terminal set has the cylindrical form (26.3). Function

$$u\,(t) = u\left(z^0\,(\,\cdot\,)\,,\ t,\ v\,(t)\right),$$

such that $v\,(\,\cdot\,) \in \Omega_V$ implies $u\,(\,\cdot\,) \in \Omega_U$ is called countercontrol stroboscopic strategy of Hajek (see [39]) of pursuer corresponding to initial state $z^0\,(\,\cdot\,)$. The game is evolving on the closed time interval $[0,\ T]$. We assume that the pursuer chooses his control in the form

$$u\,(t) = \begin{cases} u_1\left(z^0\,(\,\cdot\,)\,,\ t,\ v\,(t)\right), & t \in [0,\ t_*)\,; \\ u_2\left(z^0\,(\,\cdot\,)\,,\ t,\ v\,(t)\right), & t \in [t_*\,,\ T]\,, \end{cases}$$

where $[0,\ t_*)$ is the active interval time, $[t_*\,,\ T]$ is the passive one, and $t_* = t_*\,(v\,(\,\cdot\,))$ is the moment of switching from the Method of Resolving Functions in first interval time to the First Direct Method of L.S. Pontryagin in the second one.

We introduce set-valued mappings

$$\bar{W}\,(t,\ v) = \pi exp_\tau\{B, t\}\phi\,(U,\ v)\,,$$

$$\bar{W}\,(t) = \bigcap_{v \in V} \bar{W}\,(t,\ v)\,,$$

Condition 2 The mapping $\bar{W}\,(t) \neq \emptyset$ for all $t \geq 0$.

288 The mapping $\bar{W}$ is upper semi-continuous and therefore Borel measurable function (see [43]). Hence, there exists at least one Borelian selection $g(t)$, $g(t) \in \bar{W}(t)$ (see [43]). Denote by $G = \{g(t) : g(t) \in \bar{W}(t), \quad t \geq 0\}$ the set of all Borelian selections of the set-valued mapping $\bar{W}(t)$. For fixed $g(\cdot) \in G$ we put

$$\xi\left(t,\ z^0(\cdot),\ g(\cdot)\right) =$$
$$= \pi exp_\tau\{B, t\} z^0(-\tau) + \int_{-\tau}^{0} \pi exp_\tau\{B, t-\tau-s\}\dot{z}^0(s)\mathrm{d}s + \int_0^t g(s)\mathrm{d}s,$$

292 and consider the resolving function

$$\alpha\left(t, s, z^0(\cdot), v, g(\cdot)\right) = \sup\{\alpha \geq 0 :$$
$$\left[\bar{W}(t-\tau-s, v) - g(t-\tau-s)\right] \cap \alpha\left[M - \xi\left(t, z^0(\cdot), g(\cdot)\right)\right] \neq \emptyset\}. \tag{26.11}$$

293 The function $\alpha\left(t, s, z^0(\cdot), v, g(\cdot)\right)$ is summable for $s \in [0,\ t]$ (see [5]).

294 We introduce the function (26.5). The value $T = T\left(z^0(\cdot),\ g(\cdot)\right)$ for the initial state $z^0(\cdot)$ of the system (26.10) and some selector $g^0(\cdot) \in G$ is the guaranteed moment of capture by the pursuer of the evader according to the Method of Resolving Functions.

298 On the other hand, we set

$$P\left(z^0(\cdot),\ g(\cdot)\right)$$
$$= \min\left\{t \geq 0 : \pi exp_\tau\{B, t\} z^0(-\tau) + \int_{-\tau}^{0} \pi exp_\tau\{B, t-\tau-s\}\dot{z}^0(s)\mathrm{d}s\right.$$
$$\left.\in M - \int_0^t \bar{W}(t-\tau-s)\mathrm{d}s\right\}. \tag{26.12}$$

299 Let us show that the quantity (26.3) is the guaranteed moment of the end of the game of approach according to the First Direct Method of L.S. Pontryagin (see [42]).

302 **Theorem 26.2** *Let the conflict controlled process (26.10), (26.3) with the initial condition (26.2) satisfy Condition 2, the set M be convex, $P\left(z^0(\cdot)\right) < +\infty$, when $P\left(z^0(\cdot)\right)$ is defined by formula (26.3).*

305 *Then a trajectory of the process (26.10), (26.3) can be brought by the pursuer from $z^0(\cdot)$ to the terminal set M^* at the moment $P\left(z^0(\cdot)\right)$.*

Proof For simplicity of presentation, denote $P_0 = P\left(z^0(\cdot)\right)$. We have the following inclusion

$$\pi exp_\tau\{B, P_0\}z^0(-\tau) + \int_{-\tau}^{0} \pi exp_\tau\{B, P_0 - \tau - s\}\dot{z}^0(s)\mathrm{d}s$$
$$\in M - \int_0^{P_0} \bar{W}(P_0 - \tau - s)\mathrm{d}s.$$

Since, there exist point $m \in M$ and selection $g(\cdot) \in G$ such that

$$\pi exp_\tau\{B, P_0\}z^0(-\tau) + \int_{-\tau}^{0} \pi exp_\tau\{B, P_0 - \tau - s\}\dot{z}^0(s)\mathrm{d}s$$
$$= m - \int_0^{P_0} g(P_0 - \tau - s)\mathrm{d}s.$$

Consider the set-valued mapping

$$\begin{aligned} U(s, v) = \{u \in U \ : \ & \pi exp_\tau\{B, P_0 - \tau - s\}\phi(u, v) \\ & -g(P_0 - \tau - s) = 0\}, \quad s \in [0, P_0], \quad v \in V. \end{aligned} \tag{26.13}$$

The mapping $U(s, v)$ and selection $u(s, v) = \operatorname{lex\,min} U(s, v)$ are Borel measurable functions in its variables.

We set the pursuers control equal to

$$u(s) = u(s, v(s)), \quad s \in [0, P_0],$$

where $v(s)$, $v(s) \in V$, is an arbitrary admissible control of the evader, and it will be a Borel measurable function of time.

From the relation (26.13) with (26.3) we obtain

$$\pi z(P_0) = \pi exp_\tau\{B, P_0\}z^0(-\tau) + \int_{-\tau}^{0} \pi exp_\tau\{B, P_0 - \tau - s\}\dot{z}^0(s)\mathrm{d}s$$
$$+ \int_0^{P_0} \pi exp_\tau\{B, P_0 - \tau - s\}\phi(u(s), v(s))\mathrm{d}s = m \in M.$$

This means that $z(P_0) \in M^*$. The proof is therefore complete.

Theorem 26.3 *Let the conflict controlled process (26.10), (26.3) with the initial condition (26.2) satisfy Condition 2.*

Then the inclusion

$$\pi exp_\tau\{B,t\}z^0(-\tau)+\int_{-\tau}^{0}\pi exp_\tau\{B,t-\tau-s\}\dot{z}^0(s)ds$$
$$\in M-\int_0^t \bar{W}(t-\tau-s)ds,\quad t\geq 0,$$

holds if and only if a selection $g(\,\cdot\,)\in G$ *exists, such that* $\xi\left(t,\ z^0(\,\cdot\,),\ g(\,\cdot\,)\right)\in M$.

Proof Letting

$$\pi exp_\tau\{B,t\}z^0(-\tau)+\int_{-\tau}^{0}\pi exp_\tau\{B,t-\tau-s\}\dot{z}^0(s)\mathrm{d}s$$
$$\in M-\int_0^t \bar{W}(t-\tau-s)\mathrm{d}s.$$

There exist point $m\in M$ and selection $g(\,\cdot\,)\in G$ such that

$$\pi exp_\tau\{B,t\}z^0(-\tau)+\int_{-\tau}^{0}\pi exp_\tau\{B,t-\tau-s\}\dot{z}^0(s)\mathrm{d}s$$
$$=m-\int_0^t g(t-\tau-s)\mathrm{d}s,$$

which is equivalent to $\xi\left(t,\ z^0(\,\cdot\,)\ \ g(\,\cdot\,)\right)=m\in M$.

Using the reverse line of reasoning we come to the required result. The proof is therefore complete.

Theorem 26.4 *Let the conflict controlled process (26.10), (26.3)) with the initial condition (26.2) satisfy Condition 2, and let the set M be convex, for the given initial state* $z^0(\,\cdot\,)$ *and some selection* $g^0(\,\cdot\,)\in G$ $T=T\left(z^0(\,\cdot\,)\ ,\ g^0(\,\cdot\,)\right)<+\infty$.

Then a trajectory of the process (26.10), (26.3) can be brought by the pursuer from $z^0(\,\cdot\,)$ *to the terminal set* M^* *at the moment* T.

Proof Let $v(s)\,,\ v(s)\in V,\ s\in[0,\ T]$ be an arbitrary Borel measurable function. First, consider the case when $\xi\left(T,\ z^0(\,\cdot\,),\ g^0(\,\cdot\,)\right)\ \notin\ M$. We introduce the controlling function

$$h(t)==1-\int_0^t\alpha\left(T,\ s,\ z^0(\,\cdot\,),\ v(s),\ g^0(\,\cdot\,)\right)ds,\quad t\geq 0.$$

From the definition of time T, there exists a switching time $t_*=t_*(v(\,\cdot\,))\,,\quad 0<t_*\leq T$, such that $h(t_*)=0$.

Let us describe the rules by which the pursuer constructs his control on the so-called active and the passive parts, $[0\,,\ t_*)$ and $[t_*\,,\ T]$, respectively.

Consider the set-valued mapping

$$U_1(s,v) = \Big\{u \in U \ : \ \pi exp_\tau\{B, T-\tau-s\}\phi(u,\ v) - g^0(T-\tau-s) \\ \in \alpha\left(T,\ s,\ z^0(\cdot), v(s), g^0(\cdot)\right)\left[M - \xi\left(T, z^0(\cdot), g^0(\cdot)\right)\right]\Big\}.$$

It follows from assumptions concerning the process (26.10), (26.3) parameters, with account of properties of the resolving function, that the mapping $U_1(s,\ v)$ is a Borel measurable function on the set $[0,\ T] \times V$. Then selection

$$u_1(s,\ v) = \text{lex}\min U_1(s,\ v)$$

appears as a jointly Borel measurable function in its variables (see [43]).

The pursuer's control on the interval $[0,\ t_*)$ is constructed in the following form

$$u(s) = u_1(s,\ v(s)),$$

being a superposition of Borel measurable functions it is also Borel measurable function (see [43]).

Set

$$\alpha\left(T,\ s,\ z^0(\cdot),\ v(s),\ g^0(\cdot)\right) \equiv 0, \quad s \in [t_*,\ T].$$

Then the mapping

$$U_2(s,\ v) \\ = \left\{u \in U : \pi exp_\tau\{B, T-\tau-s\}\phi(u,\ v) - g^0(T-\tau-s) = 0\right\}, \quad s \in [t_*,\ T],\ v \in V$$

is Borel measurable function in its variables, and its selection

$$u_2(s,\ v) = \text{lex}\min U_2(s,\ v)$$

is Borel measurable function as well.

On the interval $[t_*,\ T]$ we set the pursuer's control equal to

$$u(s) = u_2(s,\ v(s)). \tag{26.14}$$

It is measurable function too.

Let $\xi\left(T,\ z^0(\cdot),\ g^0(\cdot)\right) \in M$. In this case, we choose the pursuer's control on the interval $[0,\ T]$ in the form (26.14).

Thus, the rules are defined, to which the pursuer should follow in constructing his control. We will now show that if the pursuer follows these rules in the course of the game, a trajectory of process (26.10) hits the terminal set at the time T under arbitrary admissible controls of the evader.

By virtue of Lemma 26.2, the Cauchy formula for the system (26.10) implies the representation

$$\pi z(T) = \pi exp_\tau\{B, T\}z^0(-\tau) + \int_{-\tau}^{0} \pi exp_\tau\{B, T-\tau-s\}\dot{z}^0(s)\mathrm{d}s + \int_0^T \pi exp_\tau\{B, T-\tau-s\}\phi(u(s), v(s))\mathrm{d}s. \tag{26.15}$$

First, we examine the case when $\xi\left(T, z^0(\cdot), g^0(\cdot)\right) \notin M$.

By adding and subtracting from the right-hand side of Eq. (26.15) the value $\int_0^T g^0(T-\tau-s)\,\mathrm{d}s$, one can deduce

$$\pi z(T) \in \xi\left(T, z^0(\cdot), g^0(\cdot)\right)\left[1 - \int_0^{t_*} \alpha\left(T, s, z^0(\cdot), v(s), g^0(\cdot)\right)\mathrm{d}s\right] + \int_0^{t_*} \alpha\left(T, s, z^0(\cdot), v(s), g^0(\cdot)\right) M\mathrm{d}s.$$

Since $\int_0^{t_*} \alpha\left(T, s, z^0(\cdot), v(s), g^0(\cdot)\right)\mathrm{d}s = 1$ and the set M is convex then $\pi z(T) \in M$. Then, applying the rule of the pursuer control for the case when $\xi\left(T, z^0(\cdot), g^0(\cdot)\right) \in M$, we obtain the inclusion $\pi z(T) \in M$. The proof is therefore complete.

Corollary 26.2 *Let the conflict-controlled process (26.10), (26.3) with the initial condition (26.2) satisfy Condition 2.*

Then for any initial state $z^0(\cdot)$ there exists a selection $g^0(\cdot) \in G$ such that

$$T\left(z^0(\cdot), g^0(\cdot)\right) \leq P\left(z^0(\cdot)\right).$$

The effectiveness of the Method of Resolving Functions, sufficient conditions that are easily verified, the ability to quickly build the resolution function, using the modern techniques of set-valued mappings and their selections, prove the relevance of this method for solving differential-difference games that are of great practical importance.

AQ1

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